Vivekananda College of Engineering & Technology, Puttur

[A Unit of Vivekananda Vidyavardhaka Sangha Puttur ®]

Affiliated to VTU, Belagavi & Approved by AICTE New Delhi

First Semester B.E Degree Preparatory Examination-April 2022

Calculus & Differential Equations-21MAT11

Duration: 3 hrs Max. Marks :100

Note: Answer 5 full questions choosing 1 full question from each module

Module 1

1a) With usual notation prove that $\tan\phi = r\frac{d\theta}{dr}$

b) Find the pedal equation of the polar curve $r^m = a^m \cos m\theta$

c) Find the radius of curvature for $x^3 + y^3 = 3axy$ at the point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$

OR

2a) Find the angle of intersection of the curves: $r^n \cos n\theta = a^n$ & $r^n \sin n\theta = b^n$

b) Find the radius of curvature for $x = a\left[\cos t + \log\left(\tan\left(\frac{t}{2}\right)\right)\right], y = a\sin t$

c) Derive the expression for radius of curvature for $r = f(\theta)$ in polar form

Module 2

3a) Exapand $e^{\sin x}$ as the Macalurin's series upto the terms containing x^4

b) Evaluate $\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^{1/x^2}$

c) If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$, Prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$

OR

4a) Find the extreme values of the function $x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$

b) If $u = x + y + z,\ v = y + z,\ w = z$ then evaluate $J = \frac{\partial(u,v,w)}{\partial(x,y,z)}$

c) Find the total derivative of $z = xy^2 + x^2y$ where $x = at, y = 2at$

Module 3

5a) Solve $x^3\frac{dy}{dx} - x^2y = -y^4\cos x$

b) A bottle of mineral water at a room temperature of $72^0 F$ is kept in a refrigerator where the temperature is $44^0 F$. After half an hour, water cooled to $61^0 F$. What is the temperature of the mineral water in another half an hour c)Solve for p; $\frac{dy}{dx}-\frac{dx}{dy}=\frac{x}{y}-\frac{y}{x}$

OR

6a)Solve : $(x^2+y^2+x)dx+xy\,dy=0$

b)Show that the family of parabolas $x^2=4a(y+a)$ is self orthogonal

c)Find the general and singular solutions of $xp^2+xp-yp+1-y=0$

Module 4

7a)Solve $(D^4-1)y=sinx+2$

b)Solve: $(D^3+8)y=1+2x+x^4$

c)Solve: $(D^2-2D+1)y=\frac{e^x}{x}$ by variation of parameter

OR

8a)Solve: $(2x+3)^2y^{11}-2(2x+3)y^1-12y=6x$

b)Solve: $x^2y^{11}-3xy^1+4y=(1+x)^2$

c)Solve : $(D^2+3D+2)y=\sinh(2x+3)$

Module 5

9a)Find the rank of the following matrices by elementary row transformations :

$$A=\begin{bmatrix}0 & 2 & 3 & 4\\ 2 & 3 & 5 & 4\\ 4 & 8 & 13 & 12\end{bmatrix}$$

b)Solve by Gauss elimination method :

$2x+3y-z=5,4x+4y-3z=3,2x-3y+2z=2$

c)Find the largest eigen value and the corresponding eigen vector of

$$A=\begin{bmatrix}2 & 0 & 1\\ 0 & 2 & 0\\ 1 & 0 & 2\end{bmatrix}$$ by using the power method

OR

10a)Find for what values of k the system of equations

$$x+y+z=1$$
$$x+2y+4z=k$$
$$x+4y+10z=k^2$$

possess a solution. Solve completely in each case.

b)Apply Gauss-Jordan method to solve the following system of equations

$3x_1+4x_2+5x_3=18,2x_1-x_2+8x_3=13,5x_1-2x_2+7x_3=20$

c)Solve the following system of equations by Gauss-Seidal method :

$x+y+54z=110,27x+6y-z=85,6x+15y+2z=72$